Econometrics I

Problem Set 6

1. (Wooldridge 5.3) Suppose that the model

\[ \text{pctstck} = \beta_0 + \beta_1 \text{funds} + \beta_2 \text{risktol} + u \]

satisfies the first four Gauss-Markov assumptions, where \( \text{pctstck} \) is the percentage of a workers pension invested in the stock market, \( \text{funds} \) is the number of mutual funds that the worker can choose from, and \( \text{risktol} \) is some measure of risk tolerance (larger \( \text{risktol} \) means the person has a higher tolerance for risk). If \( \text{funds} \) and \( \text{risktol} \) are positively correlated, what is the inconsistency in \( \hat{\beta}_1 \), the slope coefficient in the simple regression of \( \text{pctstck} \) on \( \text{funds} \)?

2. (Wooldridge 6.3) The following model allows the return to education to depend upon the total amount of both parents education, called \( \text{pareduc} \):

\[ \log(\text{wage}) = \beta_0 + \beta_1 \text{educ} + \beta_2 \text{educ} \cdot \text{pareduc} + \beta_3 \text{exper} + \beta_4 \text{tenure} + u \]

(a) Show that, in decimal form, the return to another year of education in this model is

\[ \Delta \log(\text{wage})/\Delta \text{educ} = \beta_1 + \beta_2 \text{pareduc}. \]

What sign do you expect for \( \beta_2 \)? Why?
(b) Using the data in WAGE2.RAW, the estimated equation is

\[
\hat{\log(wage)} = 5.65 + 0.047 \text{educ} + 0.0078 \text{educ} \cdot \text{pareduc} + 0.019 \text{exper} + 0.010 \text{tenure}
\]

\[n = 722, R^2 = 0.169.
\]

(Only 722 observations contain full information on parents education.) Interpret the coefficient on the interaction term. It might help to choose two specific values for pareduc. For example, pareduc = 32 if both parents have a college education, or pareduc = 24 if both parents have a high school education. And to compare the estimated return to educ.

(c) When pareduc is added as a separate variable to the equation, we get:

\[
\hat{\log(wage)} = 4.94 + 0.097 \text{educ} + 0.033 \text{pareduc} - 0.0016 \text{educ} \cdot \text{pareduc} + 0.020 \text{exper} + 0.010 \text{tenure}
\]

\[n = 722, R^2 = 0.174.
\]

Does the estimated return to education now depend positively on parent education? Test the null hypothesis that the return to education does not depend on parent education.

3. (Wooldridge 6.7) Let \(\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_k\) be the OLS estimates from the regression of \(y_i\) on \(x_{i1}, \ldots, x_{ik}, i = 1, 2, \ldots, n\). For nonzero constants \(c_1, \ldots, c_k\), argue that the OLS intercept and slopes from the regression of \(c_0y_i\) on \(c_1x_{i1}, \ldots, c_kx_{ik}, i = 1, 2, \ldots, n\), are given by \(\tilde{\beta}_0 = c_0\hat{\beta}_0, \tilde{\beta}_1 = \left(\frac{c_1}{c_0}\right)\hat{\beta}_1, \ldots, \tilde{\beta}_k = \left(\frac{c_k}{c_0}\right)\hat{\beta}_k\). (Hint: Use the fact that the \(\hat{\beta}_j\) solve the first order conditions in (3.13), and the \(\tilde{\beta}_j\) must solve the first order conditions involving the rescaled dependent and independent variables.)
4. Based on our discussion of individual and joint tests of hypothesis based, respectively, on the \( t \) and \( F \) tests, explain the most likely reason of the following situation.

(a) Reject the joint null on the basis of the \( F \) statistic, but do not reject each separate null on the basis of the individual \( t \) tests.

(b) Reject the joint null on the basis of the \( F \) statistic, reject one individual hypothesis on the basis of the \( t \) test and do not reject the other individual hypotheses on the basis of the \( t \) test.

(c) Reject the joint null hypothesis on the basis of the \( F \) statistic, and reject each separate null hypothesis on the basis of the individual \( t \) test.

(d) Do not reject joint null on the basis of the \( F \) statistics, and do not reject each separate null on the basis of individual \( t \) tests.

(e) Do not reject the joint null on the basis of the \( F \) statistic, reject one individual hypothesis on the basis of a \( t \) test, and do not reject the other individual hypotheses on the basis of the \( t \) test.

(f) Do not reject the joint null on the basis of the \( F \) statistic, but reject each separate null on the basis of individual \( t \) tests.

5. (Wooldridge C6.2)