1. A hollow tube is bent into a circular shape as shown in Fig. 1. The tube is open at both ends and partially filled with liquid through an arc of 180°. The tube is held fixed in a vertical plane. By pressurizing one end of the tube, the liquid is displaced through an arbitrary angle \( \theta \), and then the pressure is released. Neglecting friction, derive the equation of motion for the liquid. (25%)

![Fig. 1](image)

2. A point mass \( M \) is attached to one end of a uniform bar of mass \( m \) and length \( L \). The bar hangs by one end from a light cord, with the mass \( M \) at the top as shown in Fig. 2.

(a) Determine the point at which the bar can be struck laterally without causing an immediate acceleration of mass \( M \). (15%)

(b) Repeat part (a) if the mass \( M \) is at the lower end of the bar. (10%)

![Fig. 2](image)
3. As shown in Fig.3, each bar is of weight \( W \) and length \( L \). If \( W = 4kL \) and the spring is unstretched when \( \alpha = 90^\circ \),

(a) Determine the values of \( \alpha \) at which the system is in equilibrium. (15%)

(b) Determine whether the equilibrium positions of the system are stable or unstable. (10%)

![Fig.3](image)

4. The weight \( W = 150N \) acts at the center of the disk as shown in Fig.4. Determine the tension in the cable and the reaction force at the pin support. (25%)

![Fig.4](image)
國立中正大學九十一學年度碩士班招生考試試題
系所別：機電光整合工程研究所 科 目：物理

貳．電 學

一. (10%) Three equal charges are placed at the corners of an equilateral triangle 0.50 m on a side. What are the magnitude and the direction of the force on each charge if the charges are each –3.0 nC?

二. (15%) A spherical shell with inner radius a and outer radius b has a uniform charge Q distributed throughout its volume. Calculate the electric field for 
\[ r < a, a < r < b, \text{ and } b < r. \]

三. (10%) Two identical 100-Ω resistors are joined together as shown in Fig. P3. What is the current through each resistor (a) when the switch S is open (b) when S is closed?

![Fig. P3](image)

四. (15%) Derive the kinetic energy of a particle of charge q and mass m moving at a radius r in a cyclotron with uniform magnetic field B.

五. (10%) A 25-µF capacitor is charged to a potential of 18 V. (a) How much charge is stored on the capacitor? (b) The potential is then changed so that the charge becomes 5.9×10⁻⁷ C. What is the change in the potential?

六. (15%) Find the currents I₁, I₂, and I₃ in the network of Fig. P6.

![Fig. P6](image)

七. (12%) A 5000-Ω resistor is joined in series with a 100-µF capacitor. (a) What is the capacitive time constant of this combination? (b) If a 9.0-V battery is suddenly connected across the RC combination, how long will it take for the capacitor voltage to reach 8.0 V?

八. (13%) A radio tuning circuit is made from a 150-µH inductor connected in parallel with a variable capacitor. The capacitor is adjusted so that its capacitance is 169-pF. (a) What is the resonant frequency of the circuit? (b) What is the reactance (impedance) of the inductor? (c) What is the reactance (impedance) of the capacitor?
1. (a) What is the minimum refractive index needed for a 45° prism as the figure shown in below to exhibit total internal reflection? (b) What would be the minimum value if the prism were immersed in water? (10%)  

![Diagram of a prism](image)

2. White light is incident normally on a uniform film of water (n=1.33) lying on a glass plate (n=1.6). Find the minimum possible thickness of the film given that in the reflected light: (a) 350 nm is enhanced or (b) 350 nm is missing. (10%)  

3. Unpolarized light of intensity $I_0$ is incident on crossed polarizers. (The transmission axes are at 90°). A third polarizer plate is placed between the first two with its axis oriented at 45°. What is final transmitted intensity? (5%)  

4. In a double-slit arrangement the 3rd order bright fringe is 16 mm from the center on a screen 2 m from the slits. If the wavelength is 590 nm, find (a) the slit separation, (b) the separation between bright fringes. (10%)  

5. Suppose in a particular region of space, two light waves of different wavelengths overlap. They may be described by the functions $\psi_1 = A\sin(k_1 x)$, $\psi_2 = A\sin(k_2 x)$, where $\lambda_1 = 1\mu m$, $\lambda_2 = 400nm$. (a) What is the period of the superposition of these two waves? (i.e., what is the beat period) Is the superposition a harmonic wave? (b) Suppose the constant phase of $\psi_2$ changes so that now it is described by $\psi_1 = A\cos(k_2 x)$. Will the period of the superposition change as a result? Why or why not? (15%)
6. Consider a straight optical fiber surrounded by an incident medium of index $n_0$. The core of the fiber has an index of $n_2$ and the cladding an index of $n_1$. (a) Please find out the numerical aperture of the optical fiber. (b) If a ray strikes its wall at an angle of $\theta$, which is within the limit of the numerical aperture of the fiber. The diameter of the core of the fiber is $D$, and the total length is $L$. Show that this ray will undergo $N$ times reflections as it bounces back and forth along the fiber, and $N$ can be described as

$$N = \frac{L \sin \theta}{D (n_2^2 \sin^2 \theta) \pm 1}$$

7. How to rotate the polarization of a linear-polarized light by 90 degrees with two plane mirrors? (10%)

8. A collimated light source has two wavelength components $\lambda_1$ and $\lambda_2$. (a) If we let the light source passes through a parallel grating pair separated by a distance of $D$ and then hits a screen $L$ away from the grating pair (see figure below), what is the optical path difference between these two wavelength components? The groove spacing of the gratings is $a$, and we only consider the first order diffraction. (b) If a reflective plane mirror replaces the screen in the above case, what will happen? (15%)

9. Please design a spectrometer (an instrument which can disperse the different wavelength/frequency components and calibrate their relative intensity) using any dispersive components and other optical components you required. A simple schematic diagram and some descriptions are necessary (10%).
1. Air is compressed adiabatically from State 1: \( p_1 = 1 \text{ bar}, T_f = 300 \text{ K} \) to State 2: \( p_2 = 15 \text{ bar}, v_2 = 0.1227 \text{ m}^3/\text{kg} \). The air is then cooled at constant volume to \( T_f = 300K \). Assume ideal gas behavior with constant specific heat, \( c_p = 0.718 \text{ kJ/kgK} \), and ignore kinetic and potential energy effects.

(a) Sketch these two processes on a \( p-v \) diagram. (5%)
(b) Calculate the temperature at State 2, \( T_f = ? \) (5%)
(c) Calculate the work for the first process and the heat transfer for the second process. (10%)

2. The electronic components of a computer are cooled by air flowing through a fan mounted at the inlet of the electronics enclosure. At steady state, air enters at 20°C, 1 atm. For noise control, the velocity of the entering air cannot exceed 1.3 m/s. For temperature control, the temperature of the air at the exit cannot exceed 32°C. The electronic components and fan receive, respectively, 80 W and 18 W of electric power. Determine the smallest fan inlet diameter, in cm, for which the limits on the entering air velocity and exit air temperature are met. Assume air behaves like an ideal gas with constant specific heat of \( c_p = 1.005 \text{ kJ/kgK} \). (20%)
3. A quality of air amounting to $2.42 \times 10^3$ kg undergoes a thermodynamic cycle consisting of three processes in series.
   **Process 1-2:** constant-volume heating at $V = 0.02$ m$^3$ from $p_1 = 0.1$ MPa to $p_2 = 0.42$ MPa.
   **Process 2-3:** constant-pressure cooling
   **Process 3-1:** isothermal heating to the initial state

Employing the ideal gas model with specific heat $c_p = 1$ kJ/kgK, evaluate the change in entropy, in kJ/kg, for each process. (10%)

4. Please explain the meanings of (a) stagnation pressure, (b) Reynolds transport theorem, (c) continuity equation, (d) fully-developed pipe flow and (e) stream function (5% each)

5. The circular stream of water from a faucet flows along the gravity direction (downward), and is observed to taper from a diameter of D to d in a distance of L. Please determine the flowrate of the water Q in terms of D, d, L and g (standard gravity). Assuming the flow is steady, incompressible, inviscid, and irrotational. (15%)

6. A gas flows steadily through a duct of varying cross sectional area. If the gas density is assumed to be uniformly distributed at any cross section, show that the conservation of mass principle leads to

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

where $\rho$ is the gas density, $V$, the average speed of gas and $A$, the cross sectional area. (10%)