工程數學含「線性代數」、「微分方程」與「複變」三大部分，每部分各佔 50%，考生必須自行選擇其中兩大部分作答。

第一部份：線性代數 50%

1. Find the eigenvalues and eigenvectors of \[ A = \begin{bmatrix} 9 & -10 & 2 \\ 6 & 5 & 2 \\ 6 & -2 & 1 \end{bmatrix} \] (10%) 

2. \( f(x) = 2x \) and \( g(x) = 3 + cx \). If \( f(x) \) and \( g(x) \) are orthogonal to each other when \( 0 \leq x \leq 1 \), find the normalized functions \( \frac{f(x)}{\|f(x)\|} \) and \( \frac{g(x)}{\|g(x)\|} \) (10%) 

3. Find the eigenvalues and eigenfunctions of the following equation, \( x^2 y'' + xy' + \lambda y = 0 \), \( \lambda > 0 \), and \( y(1) = 0; \ y(\pi) = 0 \). Show that this equation satisfies the Sturm-Liouville form. (15%) 

4. The figure shows an electrical circuit. If current \( r(t) = \delta(t) \) (impulse function), please find the Laplace transformation of \( v_1(t) \) and \( v_2(t) \) (15%)
第二部份 微分方程 50%

1. Solve the following second-order non-homogeneous differential equation:
   \[ x^2 y'' - 3xy' + 4y = 12 \] (10%)

2. The figure shows an undamped spring-mass system. The excitation force is \( f(t) = \cos(2t) \). The initial conditions are \( y(0) = 0 \) and \( \frac{dy}{dt}(0) = 2 \), if \( M = 1 \) and \( K = 4 \), please find the complete solution of \( y(t) \) (15%)

![Spring-mass system diagram]

3. Solve the partial differential equation (PDE)
   \[ \frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2} \]
   subjected to the following initial and boundary conditions
   \[ f(x,0) = 0 \]
   \[ f(0,t) = 1 \]
   \[ f(1,t) = 0 \]
   either by the method of separation of variables or by the method of Laplace transformation. (25%)
第三部份 複變 50%

1. An analytic complex function \( f(z) \) is represented by 
   \[ f(z) = \phi + i \psi, \quad \text{where} \quad z = x + iy. \]
   If \( \phi = x^3 + 4x - y^2 + 2y \), find:
   (a) \( \psi = ? \) (5%)
   (b) \( f(z) = ? \) (Representing \( f(z) \) in terms of the variable \( z \)) (5%)

2. Prove that
   (a) \( \sin(x + iy) = \sin x \cosh y + i \cos x \sinh y \) (8%)
   (b) \( \frac{d\xi}{dz} \) doesn't exist anywhere, where \( \xi \) is the conjugate of \( z \). (7%)

3. For the following two functions, find the Laurent series about
   the indicated singularity and indicate the range of convergence
   for the series you obtained.
   (a) \( z \cos \frac{1}{z}; \quad z = 0 \) (5%)
   (b) \( \frac{1}{z(z+2)}; \quad z = -2 \) (5%)

4. Evaluate
   (a) \( \int_{-\infty}^{\infty} \frac{dx}{x^4 + 1} = ? \) (6%)
   (b) \( \int_{0}^{2\pi} \frac{d\theta}{5 + 3 \sin \theta} = ? \) (9%)