Mereology/Mereotopology: Development and Problems

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I. Overview

“Meros” is the Greek word for “part”. Hence intuitively mereology is a theory of “part”. More precisely, such a theory is obtained by pinning down the most general principles of the usage of the binary predicate “part”. But “part” and “whole” are closely related notions, for a statement such as “x is a part of y” might be rephrased as “y is ‘the whole’ (quite often one means by this just ‘the individual’ or ‘the object’) which contains x”. In this light, if we think that y is not a whole at all, then we probably would not say that x is one of its parts in the first place. This could explain why it is a widespread tendency to think of mereology as a theory not only of “part” but also of “whole”. Nonetheless, it turns out that mereology cannot express certain important features, such as “self-connectedness”, of a whole (For a formal proof of the inadequacy of mereology, see Chapter Four of my doctoral dissertation *The Logic and Metaphysics of Part-whole Relations*, Columbia University, 2005. Let us abbreviate it as LM henceforth), and this motivates some authors to enhance mereology by adding a new primitive which can help to give a better account. The most popular way is to add a topological term “connectedness” on top of the mereological term “part” and the resulting theory is called “mereotopology”.

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1 Some authors such as Casati and Varzi indeed preserve the original primitive “part” and enhance the theory by adding “connectedness”. But some others such as Whitehead just adopt “connectedness” as the only primitive and then define “part” accordingly. See Chapter Seven of LM for these two different approaches.
Upon quick reflection, we can see that it is somehow a natural tendency for us to analyze a thing in terms of the part-whole relations in which it is involved. As a matter of fact, we apply this kind of analyses to almost every kind of being. Consider the following examples:

(1) This cap is part of this pen.
(2) This handle is part of this cup.
(3) The left half of this chalkboard is part of the whole chalkboard.
(4) My first twenty years is the best part of my life.
(5) Being honest is part of being moral.
(6) 2 is a part of 3 (in a set-theoretical sense).

This small sample already shows how diverse the uses of “part” are: a part can be disconnected from the remainder of the whole, or connected but still conspicuous, or connected and quite arbitrarily demarcated; it can also be temporal, or otherwise abstract.

Let us call the analyses of the following kind “mereological”. Unsurprisingly, serious inquiries concerning mereological analyses have been addressed from time to time by philosophers since a long, long time ago. We can trace the origins of mereology back to ancient Greek times, with the Presocratic atomists. Then it continued to be a subject of interest through the centuries: among the authors we can find such big names as Plato, Aristotle, Thomas Aquinas, Leibniz, Kant, and so on. Nonetheless, it was only with the works of Franz Brentano and his pupils, especially Husserl’s third Logical

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Investigation (1901), that mereology emerged as a formal theory. Husserl’s work was a serious attempt to formulate mereology rigorously, but he did not disentangle purely mereological concepts from other relevant ontological notions. He remarks:

By a whole we understand a range of contents which are all covered by a single foundation without the help of further contents. The contents of such a range we call its parts. Talk of the singleness of the foundation implies that every content is foundationally connected, whether directly or indirectly, with every content.³

It is unclear what can be a “foundation” which determines a whole. One might say that it is some kind of “ontological integrity” but such a term is no less obscure than what it is intended to explain. Husserl’s thesis is profoundly controversial. However, his question about what can make a “mere aggregate” a “whole” remains a central issue in present-day philosophy of mereology.

The first formal system of what we know as mereology now is due to Lesniewski⁴. However, the first English publication about mereology is due to Leonard and Goodman⁵. Both theories were developed mostly in a nominalistic spirit (Leonard and Goodman’s theory is called, significantly, the calculus of individuals). The inception of mereology was urged by some paradoxes derived from native set theory. We shall look into the relation between mereology and set theory in the next section. Here let us just note that the first reason, which is historical, for devising a mereological theory, is that it provides an ontologically parsimonious alternative to set theory.

Philosophers’ interest in Mereology has not faded since then. But ensuing theories have been grown out of the aforementioned nominalism. Quite a few authors who are still active now (among them are Peter Simons, Barry Smith, Achille Varzi and Roberto Casati⁶) argue that mereology should be a central chapter of formal ontology, understood as a theory of the most general features of the world, whatever the entities that inhabit it. We shall mark this as the second reason of coming up with a mereological theory. Nonetheless, though mereology is intended to serve as a general tool in metaphysics, what should be the basic laws of mereology has become a matter of profound philosophical controversy. We shall introduce the main problems under debate in the third section.

It is noteworthy that much effort has been devoted to the application of mereology to geometry since the early stage⁷. In any case, ordinary objects are located in space. Thus, they can be represented by their spatial structures, and we can carry on mereological analyses of those spatial structures instead of dealing with the real things. Some meaningful properties of an object, such as its color, weight, chemical properties, and so on, will be overlooked in such a picture, but this somewhat abstract domain is still interesting. In this light, mereology may be thought of as a theory of spatial representation. Nonetheless, it is intuitive that an adequate theory of spatial representation should at least be able to discern a connected structure from a disconnected one. But, as mentioned earlier, mereology cannot do that and therefore is too weak to be a good theory of spatial representation; a common solution is to add a

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⁶ For example, Peter Simons tries to pin down those mereological principles which are analytically true. See his *Parts: A Study in Ontology*. Oxford: Clarendon Press, 1987.
topological primitive to mereology to obtain so-called mereotopology. Mereotopology has recently become popular among those researchers who are interested in providing qualitative models of the spatio-temporal world. For example, nowadays spatial reasoning is one of the central topics in AI, and mereotopological theories such as the RCC (Region Connection Calculus)\(^8\) has proved very efficient as an alternative to standard quantitative methods. Philosophically speaking, it is also important to formulate a theory to deal with the mereological analyses of three-dimensional Euclidean space, for arguably it is one of the best candidates for the representation of the commonsensical world. Thus the third important reason for devising a merelogical (mereotopological) theory is to give an account of the mereological analyses of space. But as we shall see in the third section, more puzzles will kick in then.

II. Classes, Sets and Mereological Sums

In the early stage of the development of set theory, mathematicians and philosophers were not so sure about what a “class” is. At that time, the present-day taxonomy that “class” can be divided into two disjoint subgenres, i.e. “set” and “proper class”, was not available, and the notion of “mereological sum” was sometimes subsumed under the notion of “class”. For example, Cantor, as a major founder of set theory, seemed to treat classes as mereological sums from time to time\(^9\). To understand the issue better, in the following we shall bring out the distinction between “proper class” and “set” as well as the distinction between “class” and “mereological sum”.

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The difference which we have known nowadays between a proper class and a set is that the former is too big to be a set, or more precisely, we can derive a contradiction should it be assumed as a set. The most important case in point is Russell’s paradox\(^{10}\). Let “\(\in\)” stands for membership relation, that is, by “\(x \in y\)”, we mean “\(x\) is a member of \(y\)”. The standard notation for a set is to embrace all of its members within a pair of braces (or in British English, curly brackets). For example, \(\{2, 4\}\) is the set which contains 2 and 4 as its members. More generally, in the early days, set theoreticians believed that \(\{x: \Phi(x)\}\) always defines a set (or a class; keep in mind that they made no distinction between those notions then), where \(\Phi(x)\) is any kind of description in which \(x\) occurs. For instance, \(\{x: x\) is a natural number which is divided by 2\}\) is the set of all even numbers. Hence 2 is a member of either set just mentioned. Now let “\(x \notin y\)” means “\(x\) is not a member of \(y\)”. Consider the set defined by \(x \notin x\), that is, \(\{x: x \notin x\}\). Call this set \(R\). In any case, either \(R \in R\) or \(R \notin R\) must be the case. If \(R \in R\), then by the definition of \(R\), \(R \notin R\); we have a contradiction. If \(R \notin R\), then by the definition of \(R\), \(R \in R\); we have a contradiction again. Hence \(R\) cannot exist; otherwise we will bump into a contradiction for sure. However, if we set by fiat that “\(x \in y\)” can make sense only if \(x\) is a set, then we can have a class of all sets each of which does not belong to itself but this class will not be a member of itself since it is not a set (for if it is a set, Russell’s paradox will apply again).

We can sum up the following: (1) “classes” include only “proper classes” and “sets”; (2) a member of a class must be a set\(^{11}\) (a proper class cannot be a member of any class); (3) if a class of sets cannot be a set (since a contradiction will be derived from

\(^{10}\) Actually Russell followed Cantor’s diagonal method and Zermelo claimed that he had found the same paradox earlier than Russell. Here we cannot afford to occupy ourselves with those historical trifles.

\(^{11}\) The domain of the standard set theory (ZFC) contains sets only. It is possible to have basic elements other than sets, but this is immaterial to what we are trying to clarify here.
the assumption that it is a set), then this class is a proper class. We did not say what a “set” is in the foregoing delineation. We might get into circularity if we cannot give some plausible examples of sets. Fortunately, so far as we know, any class with finite members is undoubtedly a set (that is to say, a proper class must have infinite members).

We have addressed the present-day distinction between “proper class” and “set” via introducing Russell’s paradox. What is a mereological sum then? Intuitively, it is just the thing generated by “putting all its parts together”. This sounds circular and we can explain better what it means by giving some examples. First of all, there is no mereological sum of “nothing”, for we cannot generate anything by putting nothing together. However, the set of nothing will be the empty set (the notation is \{\}); this is owing to the fact that set theory is committed to the abstract things generated by adding that “magic” pair of braces or curly brackets, so the set of nothing will be something. Now, let us start with a person, say, John. John is not equal to \{John\} since John is a member of \{John\} but is not a member of himself (it is a set theoretic principle that A is equal to B if and only if they share all the same members). That is, the set of John is not equal to John. However, the mereological sum of John is John again, for if we put all the parts of John together, the resulting object should be no more and no less than John (it is a mereological principle that A is equal to B if and only if they share all the same parts).

Consider another example. Suppose we start with two objects, say, x and y. How many sets can we generate from them? Obviously, there are infinitely many, for x is not equal to \{x\} and by the same token, \{x\} is not equal to \\{\{x\}\} and such a process can go on forever. But how many mereological sums can be generated in this case? It
is much, much fewer; there are only three: x, y and xy (the thing generated by putting x and y together) since, for example, the mereological sum of x and xy is again xy (trivially, two sums share the same parts). In general, if we start with n objects, there are only $2^n - 1$ mereological sums.

With all this, it is easier to appreciate why mereology can serve as an alternative to set theory. When Russell’s paradox was discovered, many mathematicians were deeply shocked, including Lesniewski. In his view, if “class” is understood as “mereological sum”, then since every sum is a part of itself (this is owing to a mereological principle that everything is a part of itself), Russell’s paradox will be solved immediately—the mereological sum of all things each of which is not a part of itself simply does not exist (nothing satisfies the condition that “x is not a part of x” and as mentioned earlier the mereological sum of nothing is just nothing). The resulting theory of interpreting “class” in the aforementioned way is a piece of mereology. Of course, now we know that with the distinction between “proper class” and “set”, Russell’s paradox can be solved within set theory; but without that kind of distinction on hand, Lesniewski’s move is understandable.

By the way, in contrast to set theory, mereology appears much more ontologically economic: set theory generates abstract things, i.e. sets, ad infinitum by applying the “magic” braces or curly brackets while mereology is not committed to the existence of abstracta: the whole can be just as concrete as the parts. Denying abstract set theoretical construction could give early authors of mereology a push toward nominalism. However, there is no necessary link between mereology and nominalism: mereology can apply to concrete individuals or purely abstract things and this depends on what intended models the authors have in mind.
III. Basic Principles and Problems

After giving a concise historical survey of where mereology came from, in this section we shall briefly introduce today’s mereological or mereotopological principles and the philosophical debates in which they are involved. First of all, there are three well-accepted mereological principles. They are “lexical” or “constitutive” in the sense that they constitute the essential part of the meaning of “part”. In other words, if someone disagrees with those principles, we will most likely doubt whether she/he really understands what “part” means.

(P1) Every x is a part of itself. (reflexivity)
(P2) If x is a part of y and y is a part of x, then x and y are identical. (antisymmetry)
(P3) If x is a part of y and y is a part of z, then x is a part of z. (transitivity)

As for mereotopology, usually it is generated by adding a new (topological) primitive “connectedness”, two basic principles of “connectedness” (C1 and C2) and a bridging principle between “part” and “connectedness” (C3).

(C1) Every x is connected with itself. (reflexivity)
(C2) If x is connected with y, then y is also connected with x. (symmetry)
(C3) If x is a part of y, then everything which is connected with x is also connected with y. (Bridging)

The principles listed above are uncontroversial. Let us see in the following those controversial ones and what philosophical concerns enter the scene (all of the questions below have been addressed in detail in LM).
**Extensionality Principle:** If x and y have proper parts, then they are identical if and only if they share all the same proper parts, where a proper part of x is a part of x but not equal to x (intuitively, a proper part of something is strictly smaller than that thing).

The major objection to this principle is that many objects can survive changes of their parts without changing their identities. For example, John’s left hand is one of his parts. Suppose John loses his left hand in an accident but manages to survive. We will most likely think that it is the same John standing in front of us though his left hand is gone. But the question is that this involves “identity through time” and it is unclear whether parts should be relativized to time. Another similar but more complicated story goes as follows: Consider a cat named “Tibbles”, its tail “Tail”, and the rest of its body “Tib”. Suppose that at time t, Tibbles is healthy and intact. However, at time t’, some terrible thing happens to Tibbles which costs it its tail. Fortunately, Tibbles manages to survive the loss of Tail. Surprisingly, this ordinary story turns out to imply a contradiction. The argument has four premises: (1) Tibbles ≠ Tib at t; (2) Tibbles = Tib at t'; (3) Tibbles at t = Tibbles at t'; (4) Tib at t = Tib at t'. By the transitivity of identity, we derive (5) Tibbles = Tib at t (from (2), (3) and (4)). But (5) contradicts (1). It is complicated and actually unclear why we should impute this to the extensionality principle. I have argued that it is begging the question to use this kind of examples to undermine the extensionality principle (see Chapter One of LM).

**Atomicity Principle:** Every thing is made up of atoms, where an atom is a thing without proper parts.

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This is a metaphysical claim, and there is not much we can count on so far to determine whether this is a true principle (we cannot confuse mereological atoms with atoms in physics and even if one intends to correlate the former with the latter, one should consider other well-known theories such as quantum mechanics, in which objects are “wave functions”). Some philosophers try to express their opinions about this principle, but their points are, as pointed out in LM (see Chapter Two), obscure and messy.

**Unrestricted Fusion (Composition) Principle:** For any collection of objects, there is an object which is composed exactly of those objects in that collection.

The composite thing thus generated is the mereological sum of the collection of objects in question. The main objection to this principle is that a lot of unlikely, strange composites will be added to the inventory of reality. For example, the computer on my desk and the tree outside my building will compose a genuine thing. Many philosophers find this hard to swallow and try to come up with criteria of composition to get rid of strange composites. However, as David Lewis points out:

> Doing away with queer fusions by restricting composition cannot succeed, unless we do away with too much else besides. For many respects of queerness are matters of degree. But existence cannot be a matter of degree...The fuzzy line between less queer and more queer fusions cannot possibly coincide with the sharp edge where existence gives out and nothing lies beyond. A restriction on your quantifiers, on the other hand, may be as fuzzy as you please.13

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So according to Lewis, the unrestricted fusion principle should be the case since there is no way to restrict it. There are other stances concerning the problem of composition: among them we can even find nihilism, that is, to deny the existence of composite things, or agnosticism, that is, to admit that we know nothing about composition. This topic has been intensively discussed since the 1980’s and is still under debate now (see Chapter Three of LM).

**Boundarylessness Principle:** There is no boundary.

This is a very controversial principle of mereotopology. “Boundary” is a topological notion. Since mereotopology contains a topological component on top of mereology, “boundary” can be formally defined there (see Chapter Four and Chapter Seven of LM). The main reason for us to come up with such a principle is that boundaries are “unreal”, that is, they are not three-dimensionally-extended impenetrable things. However, in ordinary language, we do talk about and actually in a sense “substantiate” boundaries from time to time. For example, it will cause enormous political tension should the president of a country claim that the boundary between his country and a neighboring country does not exist. On the other hand, we also find some touchy puzzles concerning boundaries. For instance, if a boundary exists, which one of the things demarcated by that boundary owns it? Nonetheless, the whole debate might result from mixing two different intended models together, for boundaries belong to the ideal picture of mathematics and it is problematic to take them as residents of the commonsensical world. This corresponds to what Whitehead calls “the fallacy of misplaced concreteness”\(^{14}\) (see Chapter Four of LM).

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The foregoing problems are directly related to the controversial mereological or
mereotopological principles and in this short paper we shall confine ourselves to this
context although there are further issues addressed in LM.